



UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING
Department of Electrical &
Computer Engineering



NE 112 *Linear algebra for nanotechnology engineering*

1.5 The axiomatic method

Douglas Wilhelm Harder, LEL, M.Math.

dwharder@uwaterloo.ca

dwharder@gmail.com





Introduction

- In this topic, we will
 - Introduce the axiomatic method
 - Describe Euclid's postulates (axioms) and propositions
 - Look at how we will use this in this course





The axiomatic method

- The axiomatic method is quite straight-forward:
 - Make some assumptions you cannot prove (axioms)
 - See what you can deduce, or prove, from these axioms
 - These are called theorems
 - Once a theorem is proven, it can be used in subsequent proofs
- Euclid's works are the first still-existing record of an attempt to apply the axiomatic method





Euclid's geometry

- Euclid wrote down a number of definitions:
 1. A point is that which has no part.
 2. A line is breadthless width.
 3. The ends of a line are points.and twenty others
- Euclid also wrote some *common notions*, which today we would describe as axioms:
 1. Things which equal the same thing also equal one another.
 2. If equals are added to equals, then the wholes are equal.
 3. If equals are subtracted from equals, then the remainders are equal.
 4. Things which coincide with one another equal one another.
 5. The whole is greater than the part.





Euclid's elements

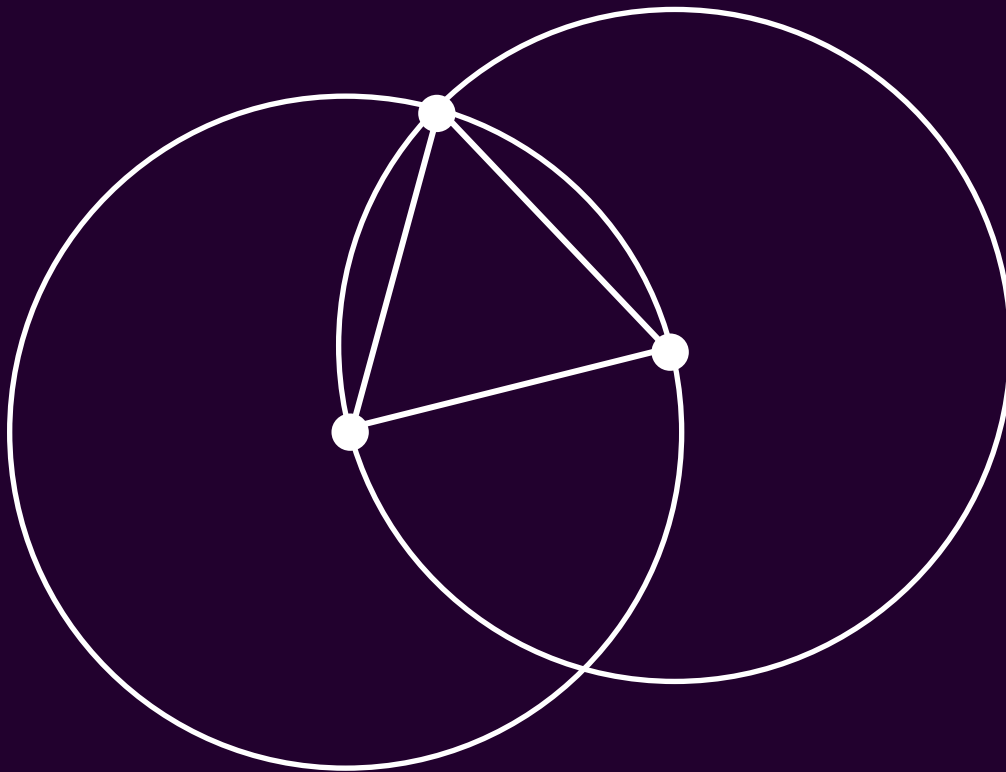
- Euclid then introduced five axioms (he called them postulates):
 1. To draw a straight line from any point to any point.
 2. To produce a finite straight line continuously in a straight line.
 3. To describe a circle with any center and radius.
 4. That all right angles equal one another.
 5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.
- From these axioms, he went on to deduce hundreds of *propositions*
 - What we now call *theorems*





Euclid's elements

- Euclid's first theorem:
 - To construct an equilateral triangle on a given finite straight line.





Axioms and theorems

- Thus, given a set of axioms,
 - we may go forward and try to prove as many theorems as possible
 - Once you prove a theorem,
 - you may use that theorem in subsequent proofs
- What is really beautiful is that if you ever find any other system that satisfies these same axioms,
 - all the theorems must also be true for that new system





Axioms and theorems

- We will start by finding those properties, or axioms, that are shared by the rational and real numbers
 - We will then introduce the complex numbers and show they satisfy these same axioms
- Following this, we will introduce the axioms of vectors
 - Those properties that are absolutely necessary
 - These will be our basis for understanding vectors and matrices
- We will see that there are other systems that satisfy these same axioms, and thus, have the same properties
 - Polynomials
 - Functions





Summary

- In this topic, you now
 - Understand the idea of an axiomatic system
 - Have reviewed a simple proof of a theorem from Euclid
 - Understand we will use this approach in this course
 - We will deduce many theorems about vector spaces and matrices
 - Have an inkling as to the idea there are more than just vectors you saw in secondary school





References

- [1] https://en.wikipedia.org/wiki/Euclid%27s_Elements
- [2] https://en.wikipedia.org/wiki/Axiomatic_system





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.





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